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$$\begin{aligned}
(p^2 + q^2)^2(m^2 + n^2)^3 &= [m(p^2 + q^2)(m^2 - 3n^2)]^2 + [n(p^2 + q^2)(3m^2 - n^2)]^2 = \\
&= [m(m^2 + n^2)(p^2 + q^2)]^2 + [n(m^2 + n^2)(p^2 + q^2)]^2 = \\
&= [m(p^2 - q^2)(m^2 - 3n^2) \pm 2npq(3m^2 - n^2)]^2 + \\
&\quad [n(p^2 - q^2)(3m^2 - n^2) \mp 2mpq(m^2 - 3n^2)]^2 = \\
&= [m(m^2 + n^2)(p^2 - q^2) \pm 2npq(m^2 + n^2)]^2 + [n(m^2 + n^2)(p^2 - q^2) \mp 2mpq(m^2 + n^2)]^2,
\end{aligned}$$

making six sets of the sum of two squares.

Substituting the respective values of p , q , m , and n , we have

$$\begin{aligned}
13^2 \cdot 61^3 &= 3042^2 + 5395^2 = 4758^2 + 3965^2 = 3810^2 + 4883^2 \\
&= 6150^2 + 733^2 = 5490^2 + 2867^2 = 1830^2 + 5917^2.
\end{aligned}$$

Solved with these six sets of values by *A. H. BELL*, and with the five sets last in order by the *PROPOSER*. Also solved by *J. H. DRUMMOND*, *C. D. SCHMITT*, and *B. F. YANNEY*.

36. Proposed by *M. A. GRUBER*, A. M., War Department, Washington, D. C.

Find the first six integral values of n in $\frac{n(n+1)}{2} = \square$.

I. Solution by Professor *J. SCHEFFER*, A. M., Hagerstown, Maryland, and *O. W. ANTHONY*, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland.

We have $n^2 + n = 2y^2$. Putting $n = \frac{t-1}{2}$, we obtain $t^2 - 8y^2 = 1$. Since $t=3, y=1$, satisfy this equation, we have $t = \frac{1}{2}[(3+2\sqrt{2})^m + (3-2\sqrt{2})^m]$, where for m successive integral numbers must be chosen. The required values of n we then obtain from the relation $n = \frac{t-1}{2}$.

For $m=1, 2, 3, 4, 5, 6$, in succession, we find in order the corresponding values of $t=3, 17, 99, 577, 3363, 19601$, and $n=1, 8, 49, 288, 1681, 9800$.

II. Solution by *A. H. BELL*, Hillsboro, Illinois.

Let $\frac{n(n+1)}{2} = \square = y^2$ say; then clearing of fractions, multiplying by 4, and adding 1 to both members, etc., $(2n+1)^2 = 8y^2 + 1 = \square = x^2$ say.

$\therefore n = \frac{x-1}{2}$. Again $x^2 - 8y^2 = 1$. The 1st convergent for the $\sqrt{8} = \frac{3}{2}$ or the solution of this celebrated equation and the value of x and y can be found, on page 53, Vol. I. of MONTHLY.

The general value for x is $x_{n+1} = 2x_1 \times x_n - x_{n-1}$, hence $x_0=1, x_1=3, x_2=6 \times 3 - 1=17, x_3=6 \times 17 - 3=99, x_4=6 \times 99 - 17=577, x_5=6 \times 577 - 99=3363$, and $x_6=19601$, etc.

$\therefore$ The required values of $n=1, 8, 49, 288, 1681, 9800$, etc.

III. Solution by the *PROPOSER*.

When $\frac{n(n+1)}{2} = \square$, one of the factors, n and $n-1$, is a square and the

other two times a square. Being known *one* of the values of n in $\frac{n(n+1)}{2} = \square$, the value next succeeding as well as the value just preceding can be found by the following formula which I deduced by inspection :

$$\frac{n(n+1)}{2} = \left(2n_1 + 1 \pm 3 \sqrt{\frac{n_1(n_1+1)}{2}}\right)^2$$

in which n_1 is a known value of n . By inspection we find that when $n=1$, $\frac{n(n+1)}{2} = \square = 1^2$. Now put $n_1=1$, and substituting in the formula, we obtain $\frac{n(n+1)}{2} = 6^2$ or 0^2 , 6^2 being the $\square$ next succeeding and 0^2 the square just preceding 1^2 . From $\frac{n(n+1)}{2} = 6^2$, we obtain $n=8 (=2 \times 2^2)$, or $-9 (= -3^2)$, and $n+1=9 (=3^2)$ or $-8 (= -2 \times 2^2)$. Now put $n_1=8$, and substituting in the formula, we get $\frac{n(n+1)}{2} = (35)^2$ or $(-1)^2$, the positive value being the next succeeding square and the negative value the one just preceding, the latter being the square with which we started. From $\frac{n(n+1)}{2} = 35^2$, we find $n=49$ or -50 , and $n+1=50$ or -49 . By continuing this process, we find the first six positive integral values of n in $\frac{n(n+1)}{2} = \square$, to be 1, 8, 49, 288, 1681, and 9800.

IV. Solution by BENJ. F. YANNEY, A. M., Professor of Mathematics, Mount Union College, Alliance, O.

Let $n=p^2$ or p^2-1 , since it must be a perfect power, or a perfect power less 1. Then $\frac{n(n+1)}{2} = \frac{p^2(p^2 \pm 1)}{2} = a^2$; whence, $p^2 \pm 1 = \frac{2a^2}{p^2} = 2q^2 \dots\dots(1)$.

Adding $2q^2 + 4pq + p^2$ to each member of equation (1), we have,
 $2q^2 + 4pq + 2p^2 \pm 1 = 4q^2 + 4pq + p^2$; or $(2q+p)^2 \pm 1 = 2(q+p)^2 \dots\dots\dots(2)$.

Since equations (1) and (2) are the same in form, if we find one set of integral values for p and q in (1), we can then readily find succeeding values by (2). Now, for $p=1$, $q=1$. $\therefore$ Other values are: 3 and 2; 7 and 5; 17 and 12; 41 and 29; 99 and 70; and so on. Then by formula $\frac{n(n+1)}{2} = \frac{p^2(p^2 \pm 1)}{2}$, the first positive integral values of n are found to be 1, 8, 49, 288, 1691, 9800.

Also solved by J. H. DRUMMOND, C. D. SCHMITT, H. C. WILKES, G. B. M. ZERR, and the PROPOSER.

ERRATA. On page 368 of December issue, line 4, for “(10+2)” read (10^m+2) ; line 9, at end, for “ B^2 ” read B_1^2 ; line 12, for “ B^2 ” read B_1^2 , and for